5. E. S. Fedorov, Protokol'naia Zapis'. *Zap. Min. Obshch.* (Official report. *Trans. Mineral. Soc.*) Vol. 26, p. 455, 1890.
6. Correspondence of E. S. Fedorov with A. Schoenflies (Published by G. B. Bokij and I. I. Schafranovskii). *Nauchnoe Nasladenie* (*Scientific Heritage*) Izdaniya Akademii Nauk SSSR (Publishing House of the Academy of Sciences USSR) Vol. 2, pp. 314–343, 1951.
7. S. A. Bogomolov, *Vyvod Pravil'nykh Sistem Po Metodu Fedorova* (*Derivation of real systems by the Fedorov method*) Part I, pp. 9–10, 1932. Leningrad, Kubutch.
8. A. Schoenflies, *Kristallsysteme und Kristallstruktur.* p. 622, Leipzig 1891.
9. Correspondence of E. S. Fedorov with P. Groth (Published by I. I. Shafranovskii, with comments), *Kristallografiia* (*Crystallography*). Sbornik Leningr. Gornogo IN-TA (Collection of Papers, Leningrad Mining Institute) No. 3, p. 219, 1955, Izd. Leningr. Gosuniversitet.
10. *E. S. Fedorov's Manuscripts*, Archives of the Academy of Sciences USSR (Ed. by I. I. Shafranovskii and N. M. Raskin) p. 140, 1957. Publ. Acad. Sci. USSR.
11. E. S. Fedorov, Pervoe Konstatirovanie Opytnym Putem Asimmorfnoi Pravil'noi Sistemy (First experimental demonstration of the asymmorphous real system) *Zap. Gornogo Instituta* (*Transactions of the Mining Institute*) Vol. 5, p. 54, 1914.
12. Ibid. p. 56.
13. Letters of T. Barker and others to E. S. Fedorov. *Kristallografiia* (*Crystallography*) (Sbornik Leningr. Gornogo IN-TA) (Collection of papers, Leningrad Mining Institute) No. 3, p. 239, 1955. Izd. Leningr. Gosuniversitet (Published by the Leningrad State University).
14. Ibid. p. 243.
15. G. N. Kovan'ko, Iz Istorii Strukturnoi Kristallografii. *Zap. Min. Obshch.* (From the history of structural crystallography. *Trans. Mineral. Soc.*) Part 87, No. 6, p. 676, 1958.
16. Letters of T. Barker and others to E. S. Fedorov. *Kristallografiia* (*Crystallography*) (Sbornik Leningr. Gornogo IN-TA) (Collection of papers, Leningrad Mining Institute) Publ. Leningrad, Gosuniversitet, pp. 249–250, No. 3, 1955.

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Artur Schoenflies

1853-1928

Schoenflies was born in the small district town of Landsberg an der Warte, then belonging to Brandenburg, now in Polish territory. He began studying mathematics in Berlin just after the war in 1870 and obtained his Ph. D. in March 1877. His main teacher was E. E. Kummer, famous for his research in geometry. The next six years Schoenflies spent as high-school teacher, the first two in Berlin, the others in Colmar in Alsace. He managed to continue research in this period along the lines begun in his thesis, combining in it geometrical inspection methods with those of analytical, synthetic and projective geometry. The success of his work led to his becoming Privatdozent

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(Lecturer) in 1884 and later (1892) Associate Professor of Applied Mathematics in Göttingen. Here he became interested in the geometrical properties of rigid-body motion which had first been studied in this sense by Camille Jordan nearly twenty years earlier. Two theorems established by Schoenflies will serve to illustrate the kind of properties involved: (i) All points of a rigid system which lie on straight lines in three of its positions belong to a (spatial) curve of the third degree; (ii) there exists a curve of the sixth degree whose points lie on circles in any four different positions of the body.—The results of these studies were incorporated in a book *Geometrie der Bewegung in synthetischer Darstellung* (Leipzig 1886) and later again summarized in an article on Kinematics in *Enzyklopaedie der mathematischen Wissenschaften*, Vol. I, section IV, 7, pg. 190–278, written together with M. Grübler.

From problems of motion, Schoenflies' attention was next drawn to those of 'plane configurations' whose study had been begun in 1887 by the Italian geometer Martinetti. A 'configuration', such as n_3, is a system of n straight lines and n points such that each line carries three points and each point is the intersection of three lines. The aim of the theory is to obtain a classification of all possible types of configurations by exploring their geometrical properties.

These by no means easy studies prepared Schoenflies well to deal with the periodic discrete groups of movement, that is, the covering operations of a space group. The earlier work on continuous groups by Jordan (1869) and on periodic discrete groups by Leonhard Sohncke (1879) Schoenflies found to be incomplete because symmetry elements of the second kind, i.e. rotation-reflection and rotation-inversion axes, were not taken into consideration. Their inclusion added 165 groups to the 65 which Sohncke had derived, bringing the total to 230. Whereas Sohncke as well as E. von Fedorov considered that the space groups, in order to be of physical significance, should be restricted by certain pre-conceived physical ideas and were otherwise not 'real', Schoenflies demanded only that the group of covering symmetry operations be geometrically possible. In this separation of pure geometry from physical statements lies the strength of his theory. Within the 'fundamental domain' (the asymmetric unit is the inadequate term now in use for its contents) 'the crystallographer may do whatever he likes', as Schoenflies puts it (*Encycl. d. math. Wiss.* l.c., pg. 468); he has full liberty there, and it is not within the province of the geometrical structure theory to impose any restrictions on the way the *Fundamentalbereich* is filled.

Schoenflies' work on structure theory began with three papers in

Mathematische Annalen (1887 and 1889), and was completed in his book *Kristallsysteme und Kristallstruktur* (Leipzig 1891). At the same time nearly identical results had been developed quite independently in St. Petersburg by E. v. Fedorov. Already earlier the latter had stressed the desirability of considering covering operations of the second kind in his book *Gestaltenlehre* (St. Petersburg 1885) and carried this idea out in the book *Symmetrie der regelmässigen Systeme von Figuren* (St. Petersburg 1890). Here, he obtained the 230 arrangements but, coming from the morphological side of crystallography, he attributed physical significance to the polyhedral fundamental domains (called by him stereohedra) and thus distinguished between the actually possible ('real') and the other 'asymmetric' space groups.

Again, in England, Lord Kelvin discussed close-packed arrangements of equal spheres and their mechanical stability (1889), and W. Barlow derived the regular packings of spheres and their symmetries (1883) and later (1891) extended this to the packing of spheres of two or three different sizes.

There was thus a sudden resurgence of interest in this problem after a stage in which it had lain dormant. Schoenflies' contribution is that of a careful mathematician who does not exceed his competence and therefore, within it, gives a final answer.

Schoenflies became deeply interested in the theory of sets, a subject then fluid and controversial, on which he wrote a famous report in 1914. He was the first author, and in numerous later editions co-author with Walter Nernst, of an introductory calculus for science students: *Einführung in die mathematische Behandlung der Naturwissenschaften*. A summary and critical review of crystallographic structure theories was given by Schoenflies in 1905 in *Enzyklopaedie der mathematischen Wissenschaften*, Vol. V, 7, pg. 437–492 (published 1922!), and in view of the increasing use of structure theory after the First World War he prepared an improved version of his older book under the title *Theorie der Kristallstruktur* (Gebr. Bornträger, Berlin, 1923).

Schoenflies became Professor of Mathematics at the University in Königsberg in 1899 and accepted in 1911 a position at the 'Academy' in Frankfurt which was about to obtain university status. He helped bringing about this transformation and became, in 1914, the first Dean of the Science Faculty. In 1920/21, the year before he retired, he was Rector of the University.—He was married and had five children.

Whoever knew Schoenflies admired his pure and considerate personality, and his modesty. He was beloved as a teacher by the students and as a colleague by the Faculty.

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